UDC 539.3

SOLUTION OF DYNAMIC PROBLEMS OF THE THEORY OF ELASTICITY FOR WEDGE-LIKE REGIONS WITH MIXED BOUNDARY CONDITIONS

PMM Vol. 42, No. 5, 1978, pp. 908-919,
V. B. PORUCHIKOV,
(Moscow)
(Received March 31, 1977)

A regular method of solving three-dimensional dynamic problems of the theory of elasticity for wedge-like regions with mixed boundary conditions is given. The mixed boundary conditions mean that a normal displacement and shear stress, or a normal stress and a tangential displacement, are specified at the boundary half-planes. The method which generalizes the result obtained by the author in [1, 2] to the case of arbitrary mixed boundary conditions combines the integral transformations with the separation of the transform singularities of the unknown functions near the edge.
A survey of the latest achievements and development of the methods of solving dynamic problems of the theory of elasticity can be found in [3].

1. Let an elastic medium with shear modulus μ and velocities of propagation of the longitudinal and transverse waves denoted by a and b, respectively, occupy the region $r > 0,\ 0 < \theta < \pi / l,\ -\infty < z < \infty$ (r, θ, and z are cylindrical coordinates), at the boundaries $\theta = 0, \pi / l$ ($^1/_2 < l,\ l \neq 1$) of which the following mixed conditions are specified:

$$w_\theta = w_\theta^k (t, r, z), \quad \sigma_{\theta r} = \sigma_{\theta r}^k (t, r, z), \quad \sigma_{\theta z} = \sigma_{\theta z}^k (t, r, z) \qquad (1.1)$$

or the conditions

$$w_r = w_r^k (t, r, z), \quad w_z = w_z^k (t, r, z), \quad \sigma_{\theta\theta} = \sigma_{\theta\theta}^k(t, r, z) \qquad (1.2)$$

where $k = 0, 1$, with the indices zero and unity referring to the boundaries $\theta = 0$ and $\theta = \pi / l$, respectively. The initial conditions are assumed to be zero; and $\mathbf{w} = \partial \mathbf{w} / \partial t = 0$ when $t = t_0$. We denote by $\mathbf{w}\ \{w_r, w_\theta, w_z\}$ the displacement vector, and by σ_{ij} the components of the stress tensor $(i, j = r, \theta, z)$.

If we express the displacement vector $\mathbf{w}$ in terms of the longitudinal and transverse scalar potentials φ, ψ_1 and ψ_2 in accordance with the formula [4, 5]

$$\mathbf{w} = \operatorname{grad} \varphi + \operatorname{rot} (\psi_1 \mathbf{e}_3) + \operatorname{rot} \operatorname{rot} (\psi_2 \mathbf{e}_3) \qquad (1.3)$$

where $\mathbf{e}_3$ is a unit vector in the direction of the z-axis, then the solutions of the dynamic problems with boundary conditions (1.1), (1.2) (we shall call them the first and second problem, respectively), can be reduced to solutions of the systems (1.4) and (1.5), respectively

$$\Delta\varphi = \frac{\partial^2 \varphi}{\partial \tau^2}, \quad \Delta\psi_j = \gamma^2 \frac{\partial^2 \psi_j}{\partial \tau^2} \quad (j = 1, 2) \qquad (1.4)$$

$$
\begin{aligned}
& w_\theta = w_\theta^\circ(\tau, r, z), \quad \sigma_{\theta r} = \sigma_{\theta r}^\circ(\tau, r, z) \\
& \sigma_{\theta z} = \sigma_{\theta z}^\circ(\tau, r, z) \quad (\theta = 0) \\
& w_\theta = w_\theta^1(\tau, r, z), \quad \sigma_{\theta r} = \sigma_{\theta r}^1(\tau, r, z) \\
& \sigma_{\theta z} = \sigma_{\theta z}^1(\tau, r, z) \quad (\theta = \pi / l) \\
& \varphi \equiv \psi_j \equiv 0 \quad (\tau < \tau_0) \\
& \Delta\varphi = \frac{\partial^2\varphi}{\partial\tau^2}, \quad \Delta\psi_j = \gamma^2 \frac{\partial^2\psi_j}{\partial\tau^2} \quad (j = 1, 2) \\
& w_r = w_r^\circ(\tau, r, z), \quad w_z = w_z^\circ(\tau, r, z) \\
& \sigma_{\theta\theta} = \sigma_{\theta\theta}^\circ(\tau, r, z) \quad (\theta = 0) \\
& w_r = w_r^1(\tau, r, z), \quad w_z = w_z^1(\tau, r, z) \\
& \sigma_{\theta\theta} = \sigma_{\theta\theta}^1(\tau, r, z) \quad (\theta = \pi / l) \\
& \varphi \equiv \psi_j \equiv 0 \quad (\tau < \tau_0)
\end{aligned} \tag{1.5}
$$

where $\tau = at$, $\tau_0 = at_0$, $\gamma = a / b > 1$, and Δ is the three-dimensional Laplace operator. In solving each of the systems (1.4) and (1.5), we must also take into account the conditions at the edge [6]

$$
\mathbf{w} = \mathbf{C} + O(r^\varepsilon), \quad \varepsilon > 0, \quad r \to 0 \quad (\mathbf{C} \equiv \mathbf{C}(\tau, z)) \tag{1.6}
$$

ensuring the integrability of the stresses near the edge and the uniqueness of the solutions of the above problems. In this manner, the solution of the first and second problem is reduced to solutions of the systems (1.4), (1.6) and (1.5), (1.6) respectively.

2. Let us solve the first problem (1.4), (1.6). We apply two-sided Laplace transforms in τ and z to the system (1.4). Then, expressing with the help of (1.3) the boundary conditions in (1.4) in terms of the longitudinal and transverse potentials and using the equations satisfied by the transverse potentials, we can show that the boundary conditions for the longitudinal and transverse potentials can be separated. As a result, the solution of the system (1.4) is reduced to solving the following three systems for $\overline{\varphi}^*$, $\overline{\psi}_1^*$ and $\overline{\psi}_2^*$:

$$
\begin{aligned}
& \Delta_1\overline{\varphi}^* = (q^2 - s^2)\overline{\varphi}^* \quad (\Delta_1 \equiv \partial^2 / \partial r^2 + r^{-1}\partial / \partial r + r^{-2}\partial^2 / \partial\theta^2) \\
& \partial\overline{\varphi}^* / \partial\theta = U_0 \quad (\theta = 0), \qquad \partial\overline{\varphi}^* / \partial\theta = U_1 \quad (\theta = \pi / l)
\end{aligned} \tag{2.1}
$$

$$
\begin{aligned}
& \Delta_1\overline{\psi}_1^* = (\gamma^2 q^2 - s^2)\overline{\psi}_1^* \\
& \overline{\psi}_1^* = V_0 \, (\theta = 0), \quad \overline{\psi}_1^* = V_1 \quad (\theta = \pi / l)
\end{aligned} \tag{2.2}
$$

$$
\begin{aligned}
& \Delta_1\overline{\psi}_2^* = (\gamma^2 q^2 - s^2)\overline{\psi}_2^* \\
& \partial\overline{\psi}_2^* / \partial\theta = W_0 \, (\theta = 0), \quad \partial\overline{\psi}_2^* / \partial\theta = W_1 \quad (\theta = \pi / l).
\end{aligned} \tag{2.3}
$$

In (2.1) —(2.3) we have

$$
U_k = r\gamma^{-2}q^{-2} \left[(\gamma^2 q^2 - s^2) dV_k / dr + s\mu^{-1} (\overline{\sigma}_{\theta z}{}^k)^* + (\gamma^2 q^2 - 2s^2)(\overline{w}_\theta{}^k)^*\right]
$$

$$V_k = (\gamma^2 q^2 - s^2)^{-1} [\mu^{-1} (\bar{\sigma}_{\theta r}^k)^* - 2d (\bar{w}_\theta^k)^* / dr]$$
$$W_k = r\gamma^{-2} q^{-2} [2s (\bar{w}_\theta^k)^* - \mu^{-1} (\bar{\sigma}_{\theta z}^k)^* + sdV_k / dr] \quad (k = 0, 1)$$

Also, the bar and the asterisk accompanying the functions f ($f = \varphi, \psi_1, \psi_2, \sigma_{\theta r}^k, \sigma_{\theta z}^k, w_\theta^k$) in (2.1) –(2.3) denote the corresponding Laplace transforms in τ and z of the function f

$$\bar{f} = \int_{-\infty}^{\infty} e^{-q\tau} f \, d\tau, \quad \bar{f}^* = \int_{-\infty}^{\infty} e^{-sz} \bar{f} \, dz$$

Here $\operatorname{Re} q > 0$ and $\operatorname{Re} s = 0$ since $f(\tau) \equiv 0$ when $\tau < \tau_0$, and we assume that $|f| < M_0 \tau^n$ as $\tau \to +\infty$ and the function $|\bar{f}|$ is integrable in z. Further, assuming that the estimate (1.6) remains valid after the application of the Laplace transforms in τ and z, we obtain

$$\bar{\mathbf{w}}^* = \text{const} + O(r^\varepsilon), \quad \varepsilon > 0, \quad r \to 0 \tag{2.4}$$

with the estimate (2.4) assumed to hold uniformly in θ.

Thus the solution of the first problem (1.4), (1.6) reduces to the solution of the system (2.1) –(2.4). The form of the system indicates that the longitudinal potential $\bar{\varphi}^*$ and transverse potentials $\bar{\psi}_1^*$ and $\bar{\psi}_2^*$ can be sought independently of each other as long as the condition (2.4) at the edge is not taken into account. In solving the systems (2.1) –(2.3) we expand, on the segment $0 \leqslant \theta \leqslant \pi / l$, the functions $\bar{\varphi}^* (q, r, \theta, s)$ and $\bar{\psi}_2^* (q, r, \theta, s)$ into the cosine series, and $\bar{\psi}_1^* (q, r, \theta, s)$ into a sine series.

We obtain the equations for the coefficients of the above expansions by multiplying the equations for $\bar{\varphi}^*$ and $\bar{\psi}_2^*$ from (2.1) and (2.3) by $2l\pi^{-1} \cos nl\theta d\theta$, and the equation for $\bar{\psi}_1^*$ from (2.2) by $2l\pi^{-1} \sin nl\theta d\theta$, and integrating with respect to θ from 0 to π / l. This yields the following second order ordinary differential equations:

$$La_n = \omega^2 a_n + f_n(r) \quad (L \equiv d^2 / dr^2 + r^{-1} d / dr - n^2 l^2 r^{-2}) \tag{2.5}$$
$$f_n(r) = 2l\pi^{-1} r^{-2} [U_0 - (-1)^n U_1]$$

$$\begin{aligned} &Lb_{nj} = \varkappa^2 b_{nj} + f_{nj}(r) \quad (j = 1, 2) \\ &f_{n1}(r) = -2l^2 n\pi^{-1} r^{-2} [V_0 - (-1)^n V_1], \quad f_{n2}(r) = \\ &\quad 2l\pi^{-1} r^{-2} [W_0 - (-1)^n W_1] \end{aligned} \tag{2.6}$$

$$\bar{\varphi}^* = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nl\theta,$$

$$a_n = \frac{2l}{\pi} \int_0^{\pi / l} \bar{\varphi}^* \cos nl\theta \, d\theta \qquad (n = 0, 1, 2, \ldots)$$

$$\bar{\psi}_1^* = \sum_{n=1}^{\infty} b_{n1} \sin nl\theta, \quad b_{n1} = \frac{2l}{\pi} \int_0^{\pi/l} \bar{\psi}_1^* \sin nl\theta \, d\theta \quad (n = 1, 2, 3, \ldots)$$

$$\bar{\psi}_2^* = \frac{b_{02}}{2} + \sum_{n=1}^{\infty} b_{n2} \cos nl\theta,$$

$$b_{n2} = \frac{2l}{\pi} \int_0^{\pi/l} \bar{\psi}_2^* \cos nl\theta \, d\theta \qquad (n = 0, 1, 2, \ldots)$$

$$\omega = (q^2 - s^2)^{1/2}, \quad \varkappa = (\gamma^2 q^2 - s^2)^{1/2}$$

We separate the branches of the functions ω and $\varkappa$ by producing cuts, in the s-plane, from the points $s = \pm q$ (for $\varkappa$ from the points $s = \pm \gamma q$) to infinity along the rays $\arg s = \arg q$ and $\arg s = \pi + \arg q$, and the branches of the radicals ω and $\varkappa$ are chosen so that $\omega = q$ and $\varkappa = \gamma q$ when $s = 0$. Then it can easily be shown that $\operatorname{Re} \omega > 0$ and $\operatorname{Re} \varkappa > 0$. Solving (2.5) and (2.6), we obtain

$$a_n = A_n K_{nl}(r\omega) + B_n I_{nl}(r\omega) + F_n(r) \tag{2.7}$$

$$F_n(r) = -K_{nl}(r\omega) \int_0^r I_{nl}(x\omega) f_n(x) x \, dx - I_{nl}(r\omega) \int_r^{\infty} K_{nl}(x\omega) f_n(x) x \, dx$$

$$b_{nj} = C_{nj} K_{nl}(r\varkappa) + D_{nj} I_{nl}(r\varkappa) + F_{nj}(r) \tag{2.8}$$

$$F_{nj}(r) = -K_{nl}(r\varkappa) \int_0^r I_{nl}(x\varkappa) f_{nj}(x) x \, dx - I_{nl}(r\varkappa) \int_r^{\infty} K_{nl}(x\varkappa) f_{nj}(x) x \, dx$$

where $I_\alpha(s)$ and $K_\alpha(s)$ are modified Bessel functions of the first and third kind, respectively.

We assume that the given functions w_θ^k, $\sigma_{\theta r}^k$ and $\sigma_{\theta z}^k$ are such that the functions f_n and f_{nj} are bounded when $r \to \infty$ and the functions rf_n and rf_{nj} behave like $\text{const} + O(r^\varepsilon)$, $\varepsilon > 0$ when $r \to 0$.

Using the fcllowing asymptotics of the cylindrical functions:

$$K_\alpha(s) \sim \sqrt{\frac{\pi}{2s}} e^{-s}, \quad I_\alpha(s) \sim \frac{1}{\sqrt{2\pi s}} e^{s}, \quad |s| \to \infty, \quad |\arg s| < \frac{\pi}{2}$$

and the boundedness of the functions f_n and f_{nj} with $r \to \infty$, we can show that the functions F_n and F_{nj} are bounded when $r \to \infty$. But in this case, if we seek the functions a_n and b_{nj} which are also bounded when $r \to \infty$ we find from (2.7) and (2.8) at once that $B_n \equiv D_{nj} \equiv 0$.

We determine the remaining coefficients A_n and C_{nj} using the condition at the edge (2.4). We expand the transforms $\bar{w}_r^*$ and $\bar{w}_z^*$, on the interval $0 \leqslant \theta \leqslant \pi / l$, into a cosine series, and $\bar{w}_\theta^*$ into a sine series using the expressions for the components of the displacement vector in terms of potentials, according to (1.3). Then we multiply the expression for the components of $\bar{w}_r^*$ and $\bar{w}_z^*$ by $2l\pi^{-1} \cos nl\theta d\theta$ and those of $\bar{w}_\theta^*$ by $2l\pi^{-1}\sin nl\theta d\theta$, and integrate then with respect to θ from 0 to π / l to obtain, from (2.4), for each n $(n = 0, 1, 2, \ldots)$, the following system of three equations:

$$\frac{da_n}{dr} + \frac{nl}{r} b_{n1} + s\frac{db_{n2}}{dr} = \text{const} + O(r^\varepsilon) \tag{2.9}$$
$$sa_n - \varkappa^2 b_{n2} = \text{const} + O(r^\varepsilon), \quad \varepsilon > 0, \quad r \to 0$$
$$-\frac{nl}{r} a_n - \frac{db_{n1}}{dr} - \frac{snl}{r} b_{n2} = \text{const} + O(r^\varepsilon)$$

which yield the coefficients A_n and C_{nj} appearing in the expressions (2.7) and (2.8) form a_n and b_{nj} (when $n = 0$, the system (2.9) degenerates into a system of two equations for a_0 and b_{02}, since $b_{01} = 0$; (2.9) utilizes the fact that rf_{n1} is bounded as $r \to 0$).

Using the asymptotic expansions of cylindrical functions with $s \to 0$

$$I_\alpha(s) = \frac{1}{\Gamma(1+\alpha)}\left(\frac{s}{2}\right)^\alpha + O(s^{2+\alpha}) \tag{2.10}$$
$$K_0(s) = -\ln s + O(1), \quad K_1(s) = s^{-1} + O(s \ln s)$$
$$K_\alpha(s) = \begin{cases} 2^{-1}\Gamma(\alpha)(2/s)^\alpha + 2^{-1}\Gamma(-\alpha)(s/2)^\alpha + O(s^{2-\alpha}) & (0 < \alpha < 1) \\ 2^{-1}\Gamma(\alpha)(2/s)^\alpha + O(s^{2-\alpha}) & (\alpha > 1) \end{cases}$$

where $\Gamma(\alpha)$ is a gamma function, and the constraints imposed on the behavior of f_n and f_{nj} with $r \to 0$, we obtain the following asymptotic expressions for $F_n(r)$ and $F_{nj}(r)$ as $r \to 0$, depending on the value of n:

$$F_0(r) = \text{const} + O(r), \quad F_1(r) = M_1 r^l + O(r) \tag{2.11}$$
$$F_n(r) = O(r) \quad (n \geqslant 2)$$
$$F_{02}(r) = \text{const} + O(r), \quad F_{1j}(r) = M_{1j} r^l + O(r)$$
$$F_{nj}(r) = O(r) \quad (n \geqslant 2)$$

$$M_1 = -\left(\frac{\omega}{2}\right)^l \frac{1}{\Gamma(1+l)} \int_0^\infty K_l(x\omega) f_1(x)\, x\, dx \quad (l < 1), \quad M_1 = 0 \quad (l > 1) \tag{2.12}$$

$$M_{1j} = -\left(\frac{\varkappa}{2}\right)^l \frac{1}{\Gamma(1+l)} \int_0^\infty K_l(x\varkappa) f_{1j}(x)\, x\, dx \quad (l < 1), \quad M_{1j} = 0 \quad (l > 1)$$

Substituting (2.7) and (2.8) into (2.9) and utilizing the asymptotic estimates (2.10) and (2.11), we find that the conditions (2.9) will hold for $n = 0$ and $n \geqslant 2$ provided that $A_n \equiv C_{nj} \equiv 0$. When $n = 1$, we have the system

$$Sr^{-l-1} + Tr^{l-1} + O(1) = \text{const} + O(r^\varepsilon)$$
$$Xr^{-l} + O(r^l) = \text{const} + O(r^\varepsilon), \quad \varepsilon > 0, \quad r \to 0$$
$$Sr^{-l-1} - Tr^{l-1} + O(1) = \text{const} + O(r^\varepsilon)$$

where

$$S = -2^{l-1}\Gamma(1+l)[A_1\omega^{-l} - C_{11}\varkappa^{-l} + sC_{12}\varkappa^{-l}]$$
$$T = -2^{-l-1}\Gamma(1-l)[A_1\omega^{l} + C_{11}\varkappa^{l} + sC_{12}\varkappa^{l}] + (M_1 + M_{11} + sM_{12})l$$
$$X = 2^{l-1}\Gamma(l)[A_1 s\omega^{-l} - \varkappa^{2-l}C_{12}]$$

The above system yields $S = 0,\ T = 0,\ X = 0$, and this gives the following expressions for A_1, C_{11} and C_{12}:

$$A_1 = \omega^l\varkappa^{2-l}s^{-1}C_{12}, \quad C_{11} = \gamma^2q^2s^{-1}C_{12} \tag{2.13}$$
$$C_{12} = \frac{2^{l+1}\sin\pi l\,\Gamma(1+l)\,s\,(M_1 + M_{11} + sM_{12})}{\pi[\omega^{2l}\varkappa^{2-l} + \varkappa^l(s^2+\gamma^2q^2)]}$$

we note that for $l > 1$ the formulas (2.13) yield $A_1 = C_{11} = C_{12} = 0$.

Thus the solution of the first problem (1.4), (1.6) in terms of the transforms has the following form $({}^1/_2 < l,\ l \neq 1)$:

$$\overline{\varphi}^* = \frac{1}{2}F_0(r) + \sum_{n=1}^{\infty}F_n(r)\cos nl\theta + \frac{\varkappa^{2-l}}{s}\omega^l\cos l\theta K_l(r\omega)C_{12} \tag{2.14}$$
$$\overline{\psi}_1^* = \sum_{n=1}^{\infty}F_{n1}(r)\sin nl\theta + \frac{\gamma^2q^2}{s}\sin l\theta K_l(r\varkappa)C_{12}$$
$$\overline{\psi}_2^* = \frac{1}{2}F_{02}(r) + \sum_{n=1}^{\infty}F_{n2}(r)\cos nl\theta + \cos l\theta K_l(r\varkappa)C_{12}$$

where the expressions for $F_n(r)$ and $F_{nj}(r)$ are given in (2.7) and (2.8), and the quantity C_{12} in (2.13).

We obtain the solution of the second problem (1.5), (1.6) in the same manner. In this case we apply Laplace transformations in τ and z to (1.5), and separate the boundary conditions for the potentials φ, ψ_1 and ψ_2 to reduce the system (1.5) to solution of the following systems for the potentials:

$$\Delta_1\overline{\varphi}^* = (q^2 - s^2)\overline{\varphi}^*, \quad \overline{\varphi}^* = U_0^\circ\ (\theta = 0), \quad \overline{\varphi}^* = U_1^\circ\ (\theta = \pi/l) \tag{2.15}$$
$$\Delta_1\overline{\psi}_1^* = (\gamma^2q^2 - s^2)\overline{\psi}_1^* \tag{2.16}$$
$$\partial\overline{\psi}_1^*/\partial\theta = V_0^\circ\ (\theta = 0), \quad \partial\overline{\psi}_1^*/\partial\theta = V_1^\circ\ (\theta = \pi/l)$$
$$\Delta_1\overline{\psi}_2^* = (\gamma^2q^2 - s^2)\overline{\psi}_2^* \tag{2.17}$$
$$\overline{\psi}_2^* = W_0^\circ\ (\theta = 0), \quad \overline{\psi}_2^* = W_1^\circ\ (\theta = \pi/l)$$

where

$$U_k^\circ = 2\gamma^{-2}q^{-2}\left[\frac{1}{2\mu}(\bar\sigma_{\theta\theta}^k)^* + s(\overline{w}_z^k)^* + \frac{d}{dr}(\overline{w}_r^k)^*\right]$$

$$V_k^\circ = r\left[(\overline{w}_r^k)^* - \frac{d}{dr}U_k^\circ - s\frac{d}{dr}W_k^\circ\right]$$

$$W_k^\circ = (s^2 - \gamma^2 q^2)^{-1}[(\overline{w}_z^k)^* - sU_k^\circ] \quad (k = 0, 1)$$

Further, expanding $\overline{\varphi}^*$ and $\overline{\psi}_2^*$ on the segment $0 \leqslant \theta \leqslant \pi / l$ into a sine series and ψ_1^* into a cosine series, we solve the system (2.15) —(2.17), with conditions (2.4) taken into account, in exactly the same manner as the first problem. As a result, the solution of the second problem (1.5), (1.6), in terms of the transforms, has the following form $(l > {}^1/_2,\ l \neq 1)$:

$$\overline{\varphi}^* = \sum_{n=1}^{\infty} F_n^\circ(r)\sin nl\theta + \frac{\varkappa^{2-l}}{s}\omega^l \sin l\theta K_l(r\omega)\, C_{12}^\circ \tag{2.18}$$

$$\overline{\psi}_1^* = \frac{1}{2}\overset{\circ}{F}_{01}(r) + \sum_{n=1}^{\infty}\overset{\circ}{F}_{n1}(r)\cos nl\theta - \frac{\gamma^2 q^2}{s}\cos l\theta K_l(r\varkappa)\, C_{12}^\circ$$

$$\overline{\psi}_2^* = \sum_{n=1}^{\infty}\overset{\circ}{F}_{n2}(r)\sin nl\theta + \sin l\theta K_l(r\varkappa)\, C_{12}^\circ$$

$$C_{12}^\circ = \frac{2^{l+1}\sin \pi l\Gamma(1+l)\, s\,(M_1^\circ - M_{11}^\circ + sM_{12}^\circ)}{\pi\,[\omega^{2l}\varkappa^{2-l} + \varkappa^l(s^2+\gamma^2 q^2)]}$$

The expressions for $F_n^\circ(r)$ and $F_{nj}^\circ(r)$ are given by the last formulas of (2.7) and (2.8), the quantities M_1° and M_{1j}° are given in (2.12). We must also replace everywhere the functions $f_n(r)$ and $f_{nj}(r)$ by $f_n^\circ(r)$ and $f_{nj}^\circ(r)$ where

$$f_n^\circ(r) = -2l^2 n\pi^{-1}r^{-2}\,[U_0^\circ - (-1)^n U_1^\circ]$$

$$f_{n1}^\circ(r) = 2l\pi^{-1}r^{-2}\,[V_0^\circ - (-1)^n V_1^\circ]$$

$$f_{n2}^\circ(r) = -2l^2 n\pi^{-1}r^{-2}\,[W_0^\circ - (-1)^n W_1^\circ]$$

Finally, we obtain the solutions of the first and second problem in terms of the potentials by finding the originals of the expressions (2.14) and (2.18) in accordance with the formulas

$$\varphi = \frac{1}{(2\pi i)^2}\int_{c_0-i\infty}^{c_0+i\infty} e^{q\tau}dq \int_{-i\infty}^{i\infty}\overline{\varphi}^* e^{sz}\,ds,$$

$$\psi_j = \frac{1}{(2\pi i)^2}\int_{c_0-i\infty}^{c_0+i\infty} e^{q\tau}dq \int_{-i\infty}^{i\infty}\overline{\psi}_j^* e^{sz}\,ds \qquad (c_0 > 0)$$

In the case of plane deformation we find that the functions $\sigma_{\theta z}^k = 0$ and $w_z^k = 0$ in systems (1.4) and (1.5), and the remaining specified functions w_ϑ^k, $\sigma_{\vartheta r}^k$, w_r^k, $\sigma_{\theta\theta}^k$, are independent of z. Expressions for the potentials $\varphi(\tau, r, \theta)$, $\psi(\tau, r, \theta)$, connected to the displacements vector by the formula

$$\mathbf{w} = \operatorname{grad} \varphi + \operatorname{rot} (\psi \mathbf{e}_3)$$

are obtained for the first and second problem from the systems (1.4), (1.5), by putting $\sigma_{\theta z}{}^k$, $w_z{}^k$, ψ_2 and all derivatives with respect to z equal to zero, and $\psi \equiv \psi_1$. As a result, we obtain the solutions of the first and second problem which, as their form implies, can be formally derived from (2.14) and (2.18) by replacing in the latter formulas $(\overline{w}_\theta{}^k)^*$, $(\overline{\sigma}_{\theta r}{}^k)^*$, $(\overline{\sigma}_{\theta z}{}^k)^*$, $(\overline{w}_r{}^k)^*$, $(\overline{\sigma}_{\theta\theta}{}^k)^*$, $(\overline{w}_z{}^k)^*$ by $\overline{w}_\theta{}^k$, $\overline{\sigma}_{\theta r}{}^k$, 0, $\overline{w}_r{}^k$, $\overline{\sigma}_{\theta\theta}{}^k$, 0, respectively, passing to the limit as $s \to 0$ and putting $\overline{\varphi} \equiv \lim \overline{\varphi}^*$ and $\overline{\psi} \equiv \lim \overline{\psi}_1{}^*$ as $s \to 0$.

Thus the solution of the plane dynamic problem with zero initial conditions (at $\tau = \tau_0$) and with boundary conditions

$$w_\theta = w_\theta{}^\circ (\tau, r), \quad \sigma_{\theta r} = \sigma_{\theta r}{}^\circ (\tau, r) \quad (\theta = 0)$$
$$w_\theta = w_\theta{}^1 (\tau, r), \quad \sigma_{\theta r} = \sigma_{\theta r}{}^1 (\tau, r) \quad (\theta = \pi / l)$$

have, in terms of the transforms, the form $(l > 1/2, \ l \neq 1)$

$$\overline{\varphi} = \frac{1}{2} F_0 (r) + \sum_{n=1}^{\infty} F_n (r) \cos nl\theta + CK_l (rq) \cos l\theta \qquad (2.19)$$

$$\overline{\psi} = \sum_{n=1}^{\infty} F_{n1} (r) \sin nl\theta + C\gamma^l K_l (r\gamma q) \sin l\theta$$

$$C = \frac{2^{l+1} \sin \pi l \Gamma (1 + l) (M_1 + M_{11})}{\pi q^l (1 + \gamma^{2l})}$$

When the boundary conditions are

$$w_r = w_r{}^\circ (\tau, r), \quad \sigma_{\theta\theta} = \sigma_{\theta\theta}{}^\circ (\tau, r) \quad (\theta = 0)$$
$$w_r = w_r{}^1 (\tau, r), \quad \sigma_{\theta\theta} = \sigma_{\theta\theta}{}^1 (\tau, r) \quad (\theta = \pi / l)$$

and initial conditions at $\tau = \tau_0$ are zero, the solution of the problem can be written in the form $(l > 1/2, \ l \neq 1)$

$$\overline{\varphi} = \sum_{n=1}^{\infty} F_n{}^\circ (r) \sin nl\theta + C^\circ K_l (rq) \sin l\theta \qquad (2.20)$$

$$\overline{\psi} = \frac{1}{2} F_{01}{}^\circ (r) + \sum_{n=1}^{\infty} F_{n1}^\circ (r) \cos nl\theta - C^\circ \gamma^l K_l (r\gamma q) \cos l\theta$$

$$C^\circ = \frac{2^{l+1} \sin \pi l \Gamma (1 + l) (M_1{}^\circ - M_{11}{}^\circ)}{\pi q^l (1 + \gamma^{2l})}$$

The functions F_n and F_{n1} and the quantities M_1 and M_{11} in (2.19) have the same form as in (2.7), (2.8) and (2.12), provided that ω is replaced by q and $\varkappa$ by γq, and

$$f_n(r) = 2l\pi^{-1}r^{-2}\,[U_0 - (-1)^n U_1],$$
$$f_{n1}(r) = -2l^2 n\pi^{-1}r^{-2}\,[V_0 - (-1)^n V_1]$$

$$U_k = r\overline{w}_\theta{}^k - \frac{1}{\gamma^2 q^2}\left(2r\frac{d^2\overline{w}_\theta{}^k}{dr^2} - \frac{d\overline{\sigma}_{\theta r}^k}{\mu dr}\right)$$
$$V_k = \frac{1}{\gamma^2 q^2}\left(\frac{\overline{\sigma}_{\theta r}^k}{\mu} - 2\frac{d\overline{w}_\theta{}^k}{dr}\right)$$

The formulas written for F_n, F_{n1}, M_1 and M_{11} remain valid for $F_n{}^\circ$, $F_{n1}{}^\circ$, $M_1{}^\circ$ and $M_{11}{}^\circ$ provided that f_n and f_{n1} are replaced by $f_n{}^\circ$ and $f_{n1}{}^\circ$ and

$$f_n{}^\circ(r) = -2l^2 n\pi^{-1}r^{-2}\,[U_0{}^\circ - (-1)^n U_1{}^\circ]$$
$$\overset{\circ}{f}_{n1}(r) = 2l\pi^{-1}r^{-2}\,[V_0{}^\circ - (-1)^n V_1{}^\circ]$$
$$U_k{}^\circ = \frac{1}{\gamma^2 q^2}\left(\frac{\overline{\sigma}_{\theta\theta}^k}{\mu} + 2\frac{d\overline{w}_r{}^k}{dr}\right)$$
$$V_k{}^\circ = r\overline{w}_r{}^k - \frac{r}{\gamma^2 q^2}\left(\frac{d\overline{\sigma}_{\theta\theta}^k}{\mu dr} + 2\frac{d^2\overline{w}_r{}^k}{dr^2}\right)$$

Finally, the originals of the solutions (2.19) and (2.20) are written in the form

$$\varphi = \frac{1}{2\pi i}\int_{c_0-i\infty}^{c_0+i\infty}\overline{\varphi}e^{q\tau}\,dq, \quad \psi = \frac{1}{2\pi i}\int_{c_0-i\infty}^{c_0+i\infty}\overline{\psi}e^{q\tau}\,dq \quad (c_0 > 0)$$

It must be remembered that in the course of solving the above problems of the dynamic theory of elasticity we required that the functions rf_n, rf_{nj}, $rf_n{}^\circ$ and $rf_{nj}{}^\circ$ be bounded when $r \to 0$. To satisfy these requirements it is sufficient that the functions, specified at the boundary, have the following asymptotic expansions as $r \to 0$:

$$(\overline{w}_\theta{}^k)^* = \sum_{j=0}^{2} d_j r^j + O(r^{2+\varepsilon}), \quad (\overline{\sigma}_{\theta r}^k)^* = d_3 + d_4 r + O(r^{1+\varepsilon}) \qquad (2.21)$$
$$(\overline{\sigma}_{\theta z}{}^k)^* = d_5 + O(r^\varepsilon), \quad \varepsilon > 0$$

for the first problem, and

$$(\overline{w}_r{}^k)^* = \sum_{j=0}^{2} d_j{}^\circ r^j + O(r^{2+\varepsilon}), \quad (\overline{w}_z{}^k)^* = d_3{}^\circ + d_4{}^\circ r + O(r^{1+\varepsilon}) \qquad (2.22)$$

$$(\bar{\sigma}_{\theta\theta}^{k})^* = d_5^\circ + d_6^\circ r + O(r^{1+\varepsilon}), \quad \varepsilon > 0$$

for the second problem, with d_j and d_j° ($j = 0-6$) independent of r.

Indeed, for the first problem the functions rf_n and rf_{n2} are in this case bounded, and the function $rf_{n1} = O(r^{-1})$, its order exceeds the admissible value. Then, differentiating in θ the equations for $\bar{\varphi}^*$, $\bar{\psi}_1^*$ and $\bar{\psi}_2^*$ appearing in the systems (2.1)–(2.3), reducing the boundary condition for $\bar{\psi}_1^*$ to the form

$$\frac{\partial^2\bar{\psi}_1^*}{\partial\theta^2}\bigg|_{\theta=0,\,\pi/l} = -r^2\left[\frac{\partial}{r\partial r}\left(r\frac{\partial\bar{\psi}_1^*}{\partial r}\right) - \varkappa^2\bar{\psi}_1^*\right]\bigg|_{\theta=0,\,\pi/l} =$$
$$-r\frac{d}{dr}\left(r\frac{dV_k}{dr}\right) + r^2\varkappa^2 V_k$$

with help of the equation for $\bar{\psi}_1^*$ and finally writing

$$\bar{\varphi}_1^* \equiv \partial\bar{\varphi}^* / \partial\theta, \; \bar{\psi}_{11}^* \equiv \partial\bar{\psi}_1^* / \partial\theta, \; \bar{\psi}_{21}^* \equiv \partial\bar{\psi}_2^* / \partial\theta$$
$$U_k^\circ \equiv U_k, \quad W_k^\circ \equiv W_k, \quad V_k^\circ \equiv -r\frac{d}{dr}\left(r\frac{dV_k}{dr}\right) + r^2\varkappa^2 V_k$$

we arrive at systems (2.15)–(2.17) for the second problem in terms of the potentials $\bar{\varphi}_1^*$, $\bar{\psi}_{11}^*$ and $\bar{\psi}_{21}^*$, the functions rf_n° and rf_{nj}° for this problem being already bounded when $r \to 0$.

Similarly, in the second problem we find that rf_{n1}° is bounded and the functions rf_n° and rf_{n2}° are of order $O(r^{-1})$ when $r \to 0$. In this case we can use exactly the same procedure to reduce the solution of the second problem to the solution of the systems (2.1)–(2.3) for the first problem (in terms of the potentials $\bar{\varphi}_1^*$, $\bar{\psi}_{11}^*$ and $\bar{\psi}_{21}^*$) provided that we write

$$\bar{\varphi}_1^* \equiv \partial\bar{\varphi}^* / \partial\theta, \quad \bar{\psi}_{11}^* \equiv \partial\bar{\psi}_1^* / \partial\theta, \quad \bar{\psi}_{21}^* \equiv \partial\bar{\psi}_2^* / \partial\theta$$
$$U_k \equiv -r\frac{d}{dr}\left(r\frac{dU_k^\circ}{dr}\right) + r^2\omega^2 U_k^\circ, \quad V_k \equiv V_k^\circ$$
$$W_k \equiv -r\frac{d}{dr}\left(r\frac{dW_k^\circ}{dr}\right) + r^2\varkappa^2 W_k^\circ$$

The functions rf_n and rf_{nj} for these systems are already bounded when $r \to 0$. Naturally, the estimates (2.21) and (2.22) and the above example of reducing one problem to the other, remain all valid in the case of plane strain (with the Laplace transform in z absent in this case).

We note that the cases $l = 1/2$ and $l = 1$ which were excluded from the discussion, follow from the results obtained in the limit as $l \to 1/2$ and $l \to 1$.

If different mixed conditions are specified at the boundaries $\theta = 0, \pi / l$,

$$w_\theta = w_\theta^\circ(t, r, z), \quad \sigma_{\theta r} = \sigma_{\theta r}^\circ(t, r, z) \qquad (2.23)$$
$$\sigma_{\theta z} = \sigma_{\theta z}^\circ(t, r, z) \quad (\theta = 0)$$
$$w_r = w_r^1(t, r, z), \quad w_z = w_z^1(t, r, z)$$
$$\sigma_{\theta\theta} = \sigma_{\theta\theta}^1(t, r, z) \quad (\theta = \pi / l)$$

then the solution of the nonstationary dynamic problem with conditions (2.13) can be represented by a superposition of the solutions of the first and second problem discussed above. Indeed, the solution of the problem in question can be written as a sum of the solutions of the problems with conditions (2.24) and (2.25)

$$\begin{aligned} &w_\theta = 0, \quad \sigma_{\theta r} = 0, \quad \sigma_{\theta z} = 0 \quad (\theta = 0) \\ &w_r = w_r^1(t, r, z), \quad w_z = w_z^1(t, r, z) \\ &\sigma_{\theta\theta} = \sigma_{\theta\theta}^1(t, r, z) \quad (\theta = \pi / l) \end{aligned} \tag{2.24}$$

$$\begin{aligned} &w_\theta = w_\theta^\circ(t, r, z), \quad \sigma_{\theta r} = \sigma_{\theta r}^\circ(t, r, z) \\ &\sigma_{\theta z} = \sigma_{\theta z}^\circ(t, r, z) \quad (\theta = 0) \\ &w_r = 0, \quad w_z = 0, \quad \sigma_{\theta\theta} = 0 \quad (\theta = \pi / l) \end{aligned} \tag{2.25}$$

But, according to the formulas for U_k, V_k and W_k, the zero boundary conditions in (2.24) at $\theta = 0$ yield $\partial\varphi / \partial\theta = \psi_1 = \partial\psi_2 / \partial\theta = 0 \ (\theta = 0)$ and (2.25) gives, with help of the formulas for U_k°, V_k° and W_k°, $\varphi = \partial\psi_1 / \partial\theta = \psi_2 = 0 \quad (\theta = \pi / l)$. In this case, extending the potentials φ and ψ_2 across the boundary $\theta = 0$ in the problem with conditions (2.24) in the even manner and the potential ψ_1 in the odd manner, we obtain the second problem for the region $|\theta| < \pi / l$:

$$\begin{aligned} &w_r = w_r^1(t, r, z), \quad w_z = w_z^1(t, r, z) \\ &\sigma_{\theta\theta} = \sigma_{\theta\theta}^1(t, r, z) \quad (\theta = \pm\pi / l) \end{aligned}$$

Similarly, extending across the boundary $\theta = \pi / l$ the potentials φ and ψ_2 in the odd manner and ψ_1 in the even manner, we reduce the problem with boundary condition (2.25) to the first problem for the region $0 < \theta < 2\pi / l$

$$\begin{aligned} &w_\theta = w_\theta^\circ(t, r, z), \quad \sigma_{\theta r} = \sigma_{\theta r}^\circ(t, r, z) \\ &\sigma_{\theta z} = \sigma_{\theta z}^\circ(t, r, z) \quad (\theta = 0, 2\pi / l) \end{aligned}$$

The above method of solving the three-dimensional nonstationary dynamic problems remains valid in the case of stationary problems, provided that we replace, in the above formulas, q by $ik + \varepsilon \ (\operatorname{Im} k = 0, \varepsilon > 0)$ and pass to the limit with $\varepsilon \to 0$. In conclusion we note, that the author used the above method to study and obtain the exact analytic solutions of the plane and three-dimensional problems of diffraction of elastic, cylindrical and spherical waves on a smooth rigid wedge of arbitrary angle [1, 2].

3. Let us carry out a direct check to see whether the expressions obtained above for the potentials, are solutions of the problems formulated. In the case of the first problem it is sufficient to show that the series expressions for the potential transforms $\bar\varphi^*$, $\bar\psi_1^*$ and $\bar\psi_2^*$ given in (2.14) satisfy the corresponding systems (2.1), (2.2) and (2.3).

We shall assume that the functions U_k, V_k and W_k are piecewise smooth.

First we shall show that when $\theta \to +0$ and $\theta \to \pi / l - 0$, then the boundary conditions specified in the systems (2.1) —(2.3) hold. We can write the expression given in (2.14) for $\bar{\psi}_1^*$ in the form

$$\bar{\psi}_1^* = \frac{\gamma^2 q^2}{s} \cdot {}_{12} \quad \sin l\theta K_l(r\varkappa) + \sum_{n=1}^{\infty} (E_{n1}{}^\circ + E_{n1}{}^1) \sin nl\theta \tag{3.1}$$

$$E_{n1}{}^\circ = -K_{nl}(r\varkappa) \int_0^{r-\varepsilon} I_{nl}(x\varkappa) f_{n1}(x)\, x\, dx - I_{nl}(r\varkappa) \int_{r+\varepsilon}^{\infty} K_{nl}(x\varkappa) f_{n1}(x)\, x\, dx$$

$$E_{n1}{}^1 = -K_{nl}(r\varkappa) \int_{r-\varepsilon}^{r} I_{nl}(x\varkappa) f_{n1}(x)\, x\, dx -$$

$$I_{nl}(r\varkappa) \int_r^{r+\varepsilon} K_{nl}(x\varkappa) f_{n1}(x)\, x\, dx \qquad (\varepsilon > 0)$$

Using now the asymptotics

$$K_\nu(r\varkappa) I_\nu(x\varkappa) \to (x / r)^\nu / (2\nu), \quad \operatorname{Re} \nu \to +\infty \tag{3.2}$$

we can show that the terms of the series for $E_{n1}{}^\circ$ decrease exponentially as $n \to \infty$, and it follows that a series containing $E_{n1}{}^\circ$ converges uniformly in $\theta \in [0, \pi / l]$. Next we pass to the limit under the summation sign as $\theta \to +0$ and $\theta \to \pi / l - 0$, and find that this series, as well as the term in (3.1) dependent on C_{12} both vanish in the limit.

It remains to inspect the limit of the series containing $E_{n1}{}^1$ as $\theta \to +0$ and $\theta \to \pi / l - 0$. Expanding the expression $x^2 f_{n1}(x) = -2\pi^{-1} l^2 n\, [V_0(x) - (-1)^n V_1(x)]$ near the point $x = r$ into a Taylor series and using the asymptotics (3.2) we find that

$$E_{n1}^1 \sim \frac{2l^2 n}{\pi} \left[\int_{r-\varepsilon}^{r} \left(\frac{x}{r}\right)^{nl} \frac{dx}{x} + \int_r^{r+\varepsilon} \left(\frac{x}{r}\right)^{-nl} \frac{dx}{x} \right] \times$$

$$\frac{V_0(r) - (-1)^n V_1(r)}{2nl} \sim \frac{2}{\pi n}$$

as $n \to \infty$. This yields

$$\sum_{n=1}^{\infty} E_{n1}^1(r) \sin nl\theta = S\left[1 + O\left(\frac{1}{n}\right)\right] \tag{3.3}$$

$$S = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin nl\theta}{n} [V_0(r) - (-1)^n V_1(r)]$$

When $\theta \to +0$ and $\theta \to \pi / l - 0$, the series for S remains the only expression yielding nonzero values. We transform this expression into

$$S = \frac{V_0(r)}{\pi i} \int_{l_0} \frac{\sin \nu (\pi - l\theta)}{\nu \sin \nu\pi} d\nu + \frac{V_1(r)}{\pi i} \int_{l_0} \frac{\sin \nu l\theta}{\nu \sin \nu\pi} d\nu$$

Here the contour l_0 (extending from $+\infty - i\varepsilon$ to $+\infty + i\varepsilon$)) forms a loop around the interval $[1, +\infty)$ and intersects the real axis at the point ν_0 $(0 < \nu_0 < 1)$.

Deforming the contour l_0 into the imaginary axis going around the pole $\nu = 0$ and taking into account the fact that the integrand functions are odd, we finally obtain

$$S = V_0(r) \frac{\pi - l\theta}{\pi} + V_1(r) \frac{l\theta}{\pi}$$

which shows that when $\theta \to +0$ and $\theta \to \pi / l - 0$, then the function $\overline{\psi}_1{}^*$ assumes the prescribed boundary values

$$\overline{\psi}_1{}^* = V_0 \quad (\theta = 0), \quad \overline{\psi}_1{}^* = V_1 \quad (\theta = \pi / l)$$

In the same manner we find that the expressions $\partial\overline{\varphi}^* / \partial\theta$ and $\partial\overline{\psi}_2{}^* / \partial\theta$ satisfy the boundary conditions specified in systems (2.1) and (2.3), respectively (differentiation with respect to θ of the series $\overline{\varphi}^*$ and $\overline{\psi}_2{}^*$ under the summation sign is justified by the fact that the resulting series in partial derivatives converge uniformly in θ for $\theta \in [\varepsilon, \pi / l - \varepsilon]$, $\varepsilon > 0$).

We shall show now that $\overline{\varphi}^*, \overline{\psi}_1{}^*$ and $\overline{\psi}_2{}^*$) given by (2.4) satisfy the differential equations in systems (2.1), (2.2) and (2.3), respectively. The series $\overline{\psi}_1{}^*$ (without the additional term contianing C_{12} which obviously satisfies the equation $(\Delta_1 - \varkappa^2) \overline{\psi}_1{}^* = 0$) can be written, with help of the Watson transformation, in the form of a contour integral

$$\sum_{n=1}^{\infty} F_{n1} \sin nl\theta = \frac{l^2}{\pi i} \int_{\pi_1} \frac{\nu \sin \nu (\pi - l\theta)}{\sin \nu\pi} E_\nu (r, \varkappa, V_0)\, d\nu + \qquad (3.4)$$

$$\frac{l^2}{\pi i} \int_{l_1} \frac{\nu \sin \nu l\theta}{\sin \nu\pi} E_\nu (r, \varkappa, V_1)\, d\nu$$

$$E_\nu (r, \varkappa, V) = K_{\nu l} (r\varkappa) \int_0^r I_{\nu l} (x\varkappa) V(x) \frac{dx}{x} + I_{\nu l} (r\varkappa) \int_r^\infty K_{\nu l} (x\varkappa) V(x) \frac{dx}{x}$$

The contour l_1 passes from the region $\operatorname{Im} \nu < 0$ to the region $\operatorname{Im} \nu > 0$ intersecting the interval $(0, 1)$ and follows the rays $\arg \nu = \pm \alpha$ $(0 < \alpha < \pi / 2)$ as $|\nu| \to \infty$.

In writing the expression (3.4) we used the estimate $E_\nu = O(\nu^{-2})$ with $\operatorname{Re} \nu \to +\infty$. The estimate can be obtained using the asymptotics (3.2). As a result, the integrand functions in (3.4) decrease exponentially in ν as $|\nu| \to \infty$ along l_1, for $\theta \in (0, \pi / l)$. Then, applying to $\overline{\psi}_1{}^*$ the differential operator $(\Delta_1 - \varkappa^2) \equiv \partial^2 / \partial r^2 + r^{-1}\partial / \partial r + r^{-2}\partial^2 / \partial\theta^2 - \varkappa^2$, we can place it under the integral signs. Remembering also that

$$(\partial^2 / \partial r^2 + r^{-1}\partial / \partial r - \nu^2 l^2 r^{-2} - \varkappa^2) E_\nu (r, \varkappa, V) = -r^{-2} V (r) \tag{3.5}$$

we find

$$(\Delta_1 - \varkappa^2) \overline{\psi}_1^* = -\frac{l^2 V_0(r)}{\pi r^2 i} \int_{l_1} \frac{\nu \sin \nu (\pi - l\theta)}{\sin \nu\pi} d\nu - \frac{l^2 V_1(r)}{\pi r^2 i} \int_{l_1} \frac{\nu \sin \nu l\theta}{\sin \nu\pi} d\nu$$

Since the integrand expressions are odd functions of ν, we can deform the contour l_1 into the imaginary axis to find that the integrals vanish and $(\Delta_1 - \varkappa^2) \overline{\psi}_1^* = 0$, Q. E. D.

Similarly we can write the series for $\overline{\varphi}^*$ (as well as for $\overline{\psi}_2^*$) of (2.14) in the form

$$\frac{1}{2} F_0 + \sum_{n=1}^{\infty} F_n \cos nl\theta = \frac{1}{2} F_0 + \frac{l}{\pi i} \int_{l_1} \frac{\cos \nu (\pi - l\theta)}{\sin \nu\pi} E_\nu (r, \omega, U_0)\, d\nu + \frac{l}{\pi i} \int_{l_1} \frac{\cos \nu l\theta}{\sin \nu\pi} E_\nu (r, \omega, U_1)\, d\nu$$

As a result, applying the operator $(\Delta_1 - \omega^2)$ to $\overline{\varphi}^*$ and taking into account (3.5) and the fact that $(\Delta_1 - \omega^2) F_0 / 2 = f_0 / 2 = l\pi^{-1} r^{-2} [U_0 (r) - U_1 (r)]$, we find the following expression for any θ form the interval $(0, \pi / l)$:

$$(\Delta_1 - \omega^2) \overline{\varphi}^* = \frac{l}{\pi r^2} [U_0(r) - U_1(r)] - \frac{l U_0(r)}{\pi i r^2} \int_{l_1} \frac{\cos \nu (\pi - l\theta)}{\sin \nu\pi} d\nu + \frac{l U_1(r)}{\pi i r^2} \int_{l_1} \frac{\cos \nu l\theta}{\sin \nu\pi} d\nu = -\frac{l U_0(r)}{\pi i r^2} \,⨍_{-i\infty}^{i\infty} \frac{\cos \nu (\pi - l\theta)}{\sin \nu\pi} d\nu + \frac{l U_1(r)}{\pi i r^2} \,⨍_{-i\infty}^{i\infty} \frac{\cos \nu l\theta}{\sin \nu\pi} d\nu = 0$$

since the integrand functions are odd in ν. An oblique stroke intersecting the integral sign indicates that, during the integration along the imaginary axis, it stands for its principal Cauchy value.

In the same manner we can show that in the case of the second problem the expressions (2.18) are solutions of systems (2.15) — (2.17).

REFERENCES

1. Poruchikov, V. B. Diffraction of a spherical elastic wave at a wedge, PMM Vol. 40, No. 5, 1976.

2. Poruchikov, V. B. Diffraction of a cylindrical elastic wave at a wedge. Izv. Akad. Nauk SSSR, MTT, No. 5, 1976.

3. Babich V. M. and Molotkov I. A. Mathematical methods of the theory of elastic waves. What is new in Science and Technology, Mechanics of Deformable Solids, Vol. 10, M., VINITI, 1977.

4. Morse P. and Feshbach, H. Methods of Theoretical Physics. McGraw-Hill, 1953.

5. Guz' A. N. and Golovchan V. T. Diffraction of Elastic Waves in Multi-connected Media. Kiev, "Nau'ova Dumka", 1972.

6. Kostrov B. V. Diffraction of a plane wave at a smooth rigid wedge in an unbounded elastic medium in absence of friction, PMM Vol. 30, No. 1, 1966.

Translated by L. K.